University of Zurich[UZH]

Institut für Politikwissenschaft

The Multilevel Logit Model for Ordinal Dependent Variables

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Part I

The Single Level Ordered Logit Model

Motivating Example

- 2001 BES respondents were asked if the statement "Labour government makes life better" described them well, badly, or neither. Thus,

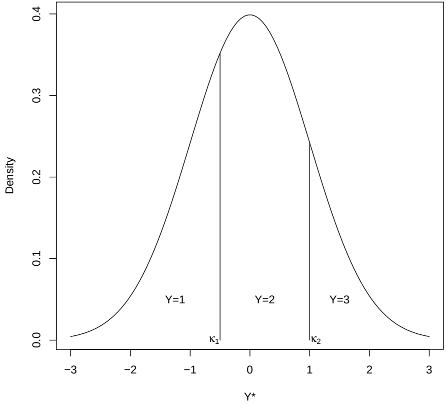
$$Y = \begin{cases} 1 & \text{Disagree} \\ 2 & \text{Neutral} \\ 3 & \text{Agree} \end{cases}$$

- We wish to model the probability of giving one or the other response.
- How do we do this?

A Latent Variable Specification

- We assume that underlying Y there exists a latent variable, Y^*, that captures the attitude with infinite precision.
- If the latent attitude exceeds a certain threshold, then the individual picks a particular response.
- The thresholds are also known as cutpoints.
- The relationship between Y, Y^*, and the cutpoints is illustrated in the next graph.

Cutpoints, Latent Attitudes, and Manifest Responses



A Model

Latent Variable

$$y_i^* = \beta_1 x_{1i} + \cdots + \beta_P x_{Pi} + \epsilon_i$$

Manifest Variable

Let $\pi_{i(m)} = \Pr(y_i > m)$, i.e., the probability that a response option greater than $m = 1, \cdots, M$ is selected. Then

$$\begin{aligned}
\pi_{i(m)} &= \Pr(y_i^* > \kappa_m) \\
&= \Pr(\beta_1 x_{1i} + \cdots + \beta_P x_{Pi} + \epsilon_i > \kappa_m) \\
&= \Pr(\epsilon_i > \kappa_m - \beta_1 x_{1i} - \cdots - \beta_P x_{Pi}) \\
&= \Pr(\epsilon_i \leq \beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_m) \\
&= F(\beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_m)
\end{aligned}$$

Common Distributions

- Ordered logit: $F(.)$ is the standard logistic distribution function.
- Ordered probit: $F(.)$ is the standard normal distribution function.

Probabilities of Specific Responses

Response Probability

Let $\pi_{im} = \Pr(y_i = m)$. Then

$$\begin{aligned}\pi_{im} &= \pi_{i(m-1)} - \pi_{i(m)} \\ &= F(\beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_{m-1}) - \\ &\quad F(\beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_m)\end{aligned}$$

with $\kappa_0 = -\infty$ and $\kappa_M = \infty$.

Example: Does Labour Make Life Better?

Parameter	Estimate	SE
Labour Identifier	1.19	0.10
Ideological Distance	-0.07	0.03
Economic Emotions	0.35	0.02
κ_1	-0.75	0.08
κ_2	1.29	0.09

Notes: $n = 2088$. Source: 2001 British Election Study. Estimated using `ologit`.

Model Identification

- Since there are only $M - 1$ unique probabilities, only $M - 1$ cutpoints can be estimated.
- This is also the reason that no constant is added to the model for Y^*.

Model Estimation

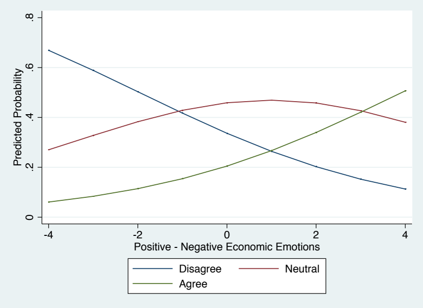
- Estimation proceeds using maximum likelihood.
- The estimators are obtained through numerical optimization, usually via Newton-Raphson.
- Note that if the model contains cutpoints only, these are adjusted so as to fit the observed response proportions.

Interpretation

Interpretation occurs via

1. Predicted probabilities
2. Cumulative odds ratios

Example: Does Labour Make Life Better?



Cumulative Odds

Cumulative Odds

$$\frac{\Pr(y_i > m)}{\Pr(y_i \leq m)} = \exp(\beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_m)$$

Cumulative Logit

Taking the natural logarithm of the cumulative odds, we obtain the cumulative logit:

$$logit_{i(m)} = \beta_1 x_{1i} + \cdots + \beta_P x_{Pi} - \kappa_m$$

Cumulative Odds Ratio

Definition

The cumulative odds ratio is the ratio of two cumulative odds, which are computed at two different values of the predictor X_p while all else remains contant.

Factor Change

For a unit change in X_p, the cumulative odds change by a factor of

$$FC = \exp(\beta_p)$$

Example: Does Labour Make Life Better?

Parameter	FC
Labour Identifier	3.30
Ideological Distance	0.93
Economic Emotions	1.41

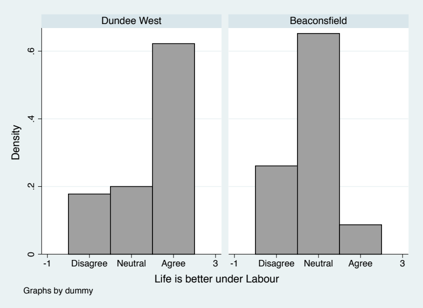
Part II

Derivation of the Multilevel Ordered Logit Model

A Motivating Example

– Often the response distribution varies over contexts.
– To accommodate this, we can add a random intercept.
– If we also expect the effect of a predictor to vary, then we may also want to add a random slope.

Example: Does Labour Make Life Better?



Random Intercept Model

Model

$$logit_{ij(m)} = \gamma_{10}x_{1ij} + \cdots + \gamma_{P0}x_{Pij} + \delta_{0j} - \kappa_m$$

Interpretation

The term $\delta_{0j} - \kappa_m$ may be viewed as a context adjusted cutpoint.

Random Intercept Model Cont'd

δ_{0j}	π_1	π_2	π_3
-1	0.500	0.318	0.182
0	0.269	0.354	0.378
1	0.119	0.258	0.622

Notes: The model is $logit_{ij(m)} = \delta_{0j} - \kappa_m$ with $\kappa_1 = -1$ and $\kappa_2 = .5$.

Estimation in Stata

```
gllamm y [x], i(name) link(ologit) nip(#) adapt
gllapred eb, u
```

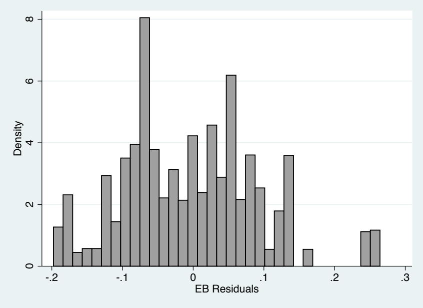
Example: Does Labour Make Life Better?

– A model with cutpoints only yields

κ_1	-1.11
κ_2	0.44
τ_{00}	0.04

– A likelihood ratio test shows evidence of random effects: $\chi^2_1 = 4.45$; the halved *p*-value is .018.

Example Cont'd



A Random Slope and Intercept Model

Model

Imagine we allow the effect of the first predictor to vary across contexts, in addition to having a random intercept. Then we specify

$$logit_{ij(m)} = \gamma_{10} x_{1ij} + \cdots + \gamma_{P0} x_{Pij} + \delta_{0j} + \delta_{1j} x_{1ij} - \kappa_m$$

A General Model With Level-2 Covariates and Cross-Level Interactions

Model

$$
\begin{aligned}
logit_{ij(m)} &= \boldsymbol{x}_{ij}^T \boldsymbol{Z}_j \gamma + \boldsymbol{x}_{ij}^T \boldsymbol{\delta}_j - \kappa_m \\
\boldsymbol{\delta}_j &\sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{T})
\end{aligned}
$$

Part III

Interpretation

Interpretation with Cumulative Odds

- Consider the random intercept and slope model

$$logit_{ij(m)} \quad = \quad \gamma_{10}x_{1ij} + \cdots + \gamma_{P0}x_{Pij} + \delta_{0j} + \delta_{1j}x_{1ij} - \kappa_m$$

- In a fixed effects interpretation, we average over i and j so that the predicted logit is $\gamma_{10}x_{1ij} + \cdots + \gamma_{P0}x_{Pij} - \kappa_m$.
- If we let the predictor X_p increase by one unit, the change in the logit is β_p.
- This means that the odds ratio changes by a factor of $\exp(\beta_p)$.

Example: Does Labour Make Life Better?

Parameter	FC
Labour Identifier	3.30
Ideological Distance	0.93
Economic Emotions	1.41

Notes: Odds ratios based on a random intercept model.

Predicted Probabilities

- Marginal probabilities average over the random effects, as they did in binary logit.
- Conditional probabilities take into consideration the level-2 variation in intercepts and/or slopes.

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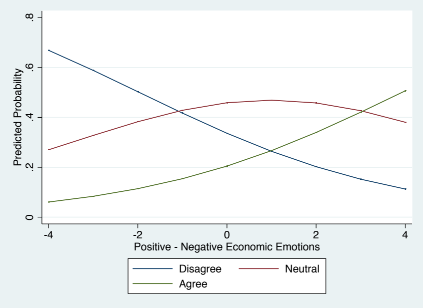
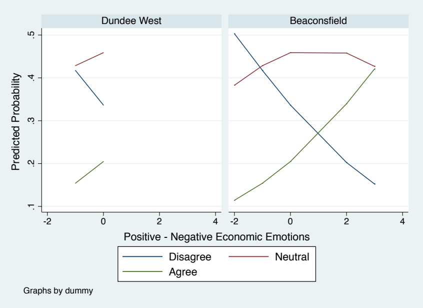
Example: Does Labour Make Life Better?



Graph with marginal predicted probabilities from a random intercept model.

Example: Does Labour Make Life Better?



Graph with conditional predicted probabilities from a random intercept model.