



The Multilevel Multinomial Logit Model

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Part I

The Single Level Model



Example: Vote Choice in Britain

- The dependent variable is

$$y_i = \begin{cases} 1 & \text{if Conservative} \\ 2 & \text{if Labour} \\ 3 & \text{if Liberal Democrat} \end{cases}$$

- We want to model the vote probability for each party.



A Behavioral Model

Utility Model

For each alternative, there is a latent utility, which consists of a systematic and random component:

$$U_{im} = V_{im} + \epsilon_{im}$$

where $m = 1, 2, \dots, M$.

Utility Maximization

A person is expected to choose alternative m if its utility exceeds that of all the remaining alternatives:

$$U_{im} > U_{ip}$$

for all $p \neq m$.



Example Cont'd

Person i is expected to vote for the Conservatives if (and only if):

$$U_{i1} > U_{i2} \text{ and } U_{i1} > U_{i3} \Leftrightarrow \\ \epsilon_{i2} < V_{i1} - V_{i2} + \epsilon_{i1} \text{ and } \epsilon_{i3} < V_{i1} - V_{i3} + \epsilon_{i1}$$



Choice Probabilities

Choice Probability

The probability of choosing alternative m is

$$\pi_{im} = \Pr(\epsilon_{ip} < V_{im} - V_{ip} + \epsilon_{im})$$

A Formula

Assuming the errors are independent and individually Gumbel distributed, it can be shown that

$$\pi_{im} = \frac{\exp V_{im}}{\sum_q \exp V_{iq}}$$



Example Cont'd

The probability of someone voting for the Conservatives is

$$\pi_{i1} = \frac{\exp V_{i1}}{\exp V_{i1} + \exp V_{i2} + \exp V_{i3}}$$



A Simple Model for V

Model

Let X be an attribute of the voter, then

$$V_{im} = \beta_{0m} + \beta_{1m}X_i$$

Thus, the effect of the attribute is allowed to vary across the alternatives.



Model Identification

- There are only $M - 1$ unique choice probabilities.
- Therefore, only $M - 1$ sets of parameters are identified.
- This means that one set of parameters is set to 0 for one of the alternatives, which then becomes the baseline.



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Part II

A Multilevel Extension



Example: Vote Choice in Britain

- Voters are nested in constituencies, whose effect we may want to consider.
- Hence, y_{ij} is voter i 's choice in constituency j .
- The systematic utility component then is V_{ijm} .
- At a minimum, we may wish to allow the intercept in this component to vary.



A Random Intecept Model

Utility Model

$$V_{ijm} = \beta_{0m} + \beta_{1m}x_{ij} + \delta_{0j}$$